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The ability to systematically control dominant nonlinear effects in the evolution of complex dynamical systems is an important research goal, with applications in several existing and emerging DOD research and development programs. The research we describe here is aimed at the development of a control methodology for lumped and distributed parameter systems, similar in scope and applicability to classical automatic control design for lumped linear systems. This program is aimed at the systematic development of feedback design methodologies for shaping, or at least influencing, the response of a broad class of nonlinear systems.

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1 Abstract

The ability to systematically control dominant nonlinear effects in the evolution of complex dynamical systems is an important research goal, with applications in several existing and emerging DOD research and development programs. The research we describe here is aimed at the development of a control methodology for lumped and distributed parameter systems, similar in scope and applicability to classical automatic control design for lumped linear systems. This program is aimed at the systematic development of feedback design methodologies for shaping, or at least influencing, the response of a broad class of nonlinear systems.

2 Proposed Research Objectives

The main objectives of this three year research effort were concerned with the control of nonlinear lumped and distributed parameter systems. This program was aimed at the systematic development of feedback design methodologies for shaping, or at least influencing, the response of a broad class of nonlinear systems.

Our optimism in pursuing this research program was based on a general maturing of the field of nonlinear control, including a remarkable spectrum of results on feedback stabilization and in shaping the response of lumped nonlinear systems. Our first explicit research task was concerned with the solution of the problem of designing feedback controllers achieving the important control objectives of asymptotic tracking and disturbance attenuation.

Task 2.1 *The formulation and derivation of conditions implying the well-posedness of the output regulator problem for nonlinear control systems.*

In the standard formulation of the output regulator problem, finite dimensional models for the signal and for the disturbance generators are assumed known. While this is typically the case for certain classes of signals to be tracked, it can not always be assumed for disturbance signals. For this reason, we refined Task 2.1 to include robustness against unstructured uncertainties, leading to the formulation of a robust disturbance attenuation problem.

Task 2.1(bis) *The formulation and derivation of conditions characterizing the existence of a robust regulator for a nonlinear system.*

The next task extended Task 2.1 in a different, but equally important, way representing the development of a nonequilibrium theory of output regulation.

Task 2.2 *The development of a nonlinear regulator theory valid globally or at least on a priori given bounded neighborhoods W of $(0,0)$ in $R^n \times R^\ell$.*

The next task dealt with the development of a theory of output regulation which would enable tracking and disturbance attenuation for broader classes of signal generators exhibiting more complicated nonlinear behavior.

Task 2.3 *The development of a regulator theory valid for tracking or rejecting signals generated by exosystems which are not necessarily "neutrally stable", including, for example, exosystems which have stable limit cycles, invariant tori, etc.*

Task 2.4 is focused on the development of a global output regulator using dissipation of energy, or passivity, as the underlying stability mechanism.

Task 2.4 *The formulation and derivation of conditions implying the existence of a solution of the tracking problem, and the nonlinear regulator problem for a plant which is feedback equivalent to a passive system.*

From a feedback design point of view, beyond the rigorous solution of the nonlinear regulator problem, a very important component of the proposed research was the actual form of the feedback compensation and whether it would be computationally tractable. In the linear case, the structure of the feedback law is that of a proportional error compensator where the feedback "gains" can be computed "off-line" by solving a Sylvestor equation, quite similar to very important roles played by the Riccati equation in linear quadratic regulator design. This fact motivated our next series of tasks dealing with the solution of nonlinear optimal control problems by feedback laws which would involve the "off-line" solution of nonlinear PDE's. Our preliminary calculations indicated that an existence theory could be developed for such equations in the context of Lagrangian analysis.

Task 2.5 *Using the methodology of Lagrangian distributions, develop sufficient conditions* *for the existence of an "off-line" characterization of globally defined optimal feedback control laws for the finite horizon regulator problem.*

Our next explicit research task focused on the application of Lagrangian methods to infinite time horizon problems.

Task 2.7 *The development of existence criteria and "off-line" characterizations of global optimal feedback laws for the infinite time optimal regulation problem.*

An important part of our effort in lumped nonlinear feedback control was concerned with the design of nonlinear feedback laws which stabilize a given nonlinear control system. We have developed a program aimed at establishing a design methodology for asymptotic stabilization of continuous-time, affine nonlinear systems in the large, based on the nonlinear dynamics concept of system transmission zeros – the notion of zero dynamics. The next task focused on an extension of this concept and technique to the class of nonaffine systems.

Task 2.8 *The development of zero dynamics for general nonlinear systems, including existence theorems and algorithms for computing nonlinear zero dynamics.*

Zero dynamics methods typically have been developed under some tacit regularity assumptions. The final two explicit research tasks for lumped nonlinear systems focused on the development and extension of this methodology to singular problems.

Task 2.9 *The development of singularly perturbed models for the zero dynamics of multivariable nonlinear systems having nonuniform (vector) relative degree, and the design of feedback laws achieving stabilization, asymptotic tracking or disturbance rejection for systems having non-uniform relative degree.*

Task 2.10 *We propose a systematic development of singular and/or global problems in noninteracting control based on the use of methods from the theory of singularities, e.g. "blowing up," together with the use of global differential geometric methods, e.g. from the emerging theory of noncommutative differential geometry, to determine solvability for arbitrary foliations Δ.*

Task 3.1 and Task 3.2 focus on one of our longer term goals in the second part of this research project, the development of a systematic feedback design methodology for nonlinear distributed parameter systems which would retain some of the intuitive appeal of classical automatic control in a spirit similar to the program pursued for the control of lumped nonlinear systems.

Task 3.1 *The development of a systematic feedback design methodology for the stabilization of minimum phase nonlinear distributed parameter systems. In particular, the development of the concept of zero dynamics and of "instantaneous gain" for nonlinear distributed parameter systems and of a feedback design theory based on asymptotic properties of the zero dynamics.*

Task 3.2 *The development of a root-locus design methodology for linear, parabolic distributed parameter systems. In particular, the development of concepts of instantaneous gain and the use of zero dynamics to design asymptotically stabilizing feedback laws for minimum phase systems.*

Another of our long term goals is described in Task 3.3 in which we proposed the development of a design methodology capable of shaping the response of distributed parameter systems. In particular, we are interested in the problem of output regulation for distributed parameter systems in the sense described above for lumped nonlinear systems. As in this case, we expect the main ingredients in the solution of this problem to be the design of stabilizing feedback laws and the development of a theory of steady state response for stable distributed parameter systems. The research described so far is aimed at the solution of the first problem. For a class of linear parabolic systems we have been able to analyze the steady state response of an asymptotically stable system. Based on this work, we expect to be able to solve the problem of output regulation for this class of control systems and for a linear exogenous system. In the nonlinear case, the development of a theory of steady state response is known to rely quite heavily on center manifold theory, which is still available in infinite dimensions and would apply whenever the exogenous signal generator is finite dimensional. This problem formulation would be of particular interest for flow control, either in the case where a reduced order finite dimensional model for the relevant features of a fluid flow is available or in the case where a finite dimensional attractor or inertial manifold for the flow dynamics is known to exist.

Task 3.3 *The development of a theory of output regulation for distributed parameter systems and its application to problems of asymptotic tracking and disturbance attenuation. In particular, the development of methods for the analysis of steady state response of stable nonlinear systems to signals produced by lumped nonlinear exogeneous systems.*

Because of its importance as a fundamental stability mechanism for lumped nonlinear systems, in the next task we proposed the rigorous formulation of the concept of dissipation of energy for input-output systems.

Task 3.4 *To develop intrinsic characterizations of those distributed parameter systems which are dissipative or passive. In particular, for linear systems to derive criteria for dissipativity in terms of positive reality of the transfer function and in general to establish the Kalman-Yacubovitch-Popov Lemma for distributed parameter systems.*

Our final explicit research task was the understanding of a more robust version of feedback linearization, viz. feedback equivalence to a passive system.

Task 3.5 *The development of conditions for feedback equivalence of a distributed parameter system to a passive or dissipative system. Moreover, the derivation and the computation of explicit feedback laws rendering a given system dissipative.*

3 Status of the Research Effort

The ability to systematically control dominant nonlinear effects in the evolution of complex dynamical systems is an important research goal, with applications in several existing and emerging DOD research and development programs. The research we describe here is aimed at the development of a control methodology for lumped and distributed parameter systems, similar in scope and applicability to classical automatic control design for lumped linear systems. Accordingly, there have been two closely allied themes which we have pursued, having their origin in two approaches to the control of complex dynamical systems – the analysis and control of as accurate a system description as possible, and the alternative development of reduced order models, which capture some of the dominant dynamical effects and require the consequent development of high performance controllers which either are more robust with respect to unmodeled dynamics or which can adapt to unknown parameter fluctuations in the reduced order model.

The main objectives of this three year research effort are concerned with the control of nonlinear lumped and distributed parameter systems. This program is aimed at the systematic development of feedback design methodologies for shaping, or at least influencing, the response of a broad class of nonlinear systems. Our optimism in pursuing this research program is based on a general maturing of the field of nonlinear control, including a remarkable spectrum of results on feedback stabilization and the application of geometric methods to the unanticipated solution of the problem of designing feedback controllers achieving the important control objectives of asymptotic tracking and disturbance attenuation.

An important part of this research effort in nonlinear feedback control is concerned with the design of nonlinear feedback laws which stabilize a given nonlinear control system, either about an equilibrium or an attractor. For the equilibrium theory, we have recently made significant progress in establishing a design methodology for asymptotic stabilization of continuous time systems in the large [5, 6], based on the nonlinear dynamics concept of system transmission zeros – the notion of zero dynamics and its relationship to more classical notions such as passivity. In [29], the zero dynamics approach is applied to develop rigorous underpinnings for a stabilization scheme which is widely used in the commercial aerospace industry – the method of inner loop - outer loop design. A more complete treatment of this development will be available in the forthcoming Washington University Ph.D. thesis by John Roltgen.

Interest in the development of a systematic methodology for nonequilibrium stabilization is becoming more widely appreciated as the benefits to taking advantage of dominant nonlinear effects have become more apparent. Examples include the use of nonlinear feedback gains to produce or to attenuate the effects of limit cycles. In [27], which has recently won the 1993 IFAC Automatica Best Paper Award, an enhancement of the geometric methodology developed for the equilibrium case is applied to the derivation of feedback laws to induce a stable limit cycle in the center manifold of a rigid spacecraft system which cannot be asymptotically stabilized about a reference equilibrium, even by nonlinear feedback. There are also interesting applications of such techniques to eliminating, or at least attenuating, hysteresis in rotating compressor stall. A nonlinear control law attenuating the effects of this hysteresis loop has been designed in ([3], [4]) using the nonlinear terms to shape the flow on a center manifold by taking advantage of a limit cycle which is produced through a Hopf bifurcation.

As an initial result in this research effort we have recently derived a theorem characterizing the existence of globally asymptotically stable compact attractors in terms of feedback equivalence to passive systems, for a suitable choice of system output. This result promises to have interesting corollaries extending our previous application involving feedback stabilization of limit cycles, as well as for more general attractors. There are of course many situations of interest, e.g tracking and regulation, where the system output (e.g., position or tracking error) cannot be treated, or redefined, as a design parameter but rather needs to controlled to a given value. In this case, passivity constructions cannot be used directly if the relative degree of the system output is higher, as is typically the case for the position variables for a mechanical system. In such cases, we would still like to take advantage of any compact attractors in, or Lyapunov stability of, the zero dynamics determined by the desired output constraint for the purpose of designing a locally asymptotically stabilizing law for a compact attractor. Here, however, examples suggest that uniform stability of the attractor may not always obtain and hence Lyapunov functions for the attracting set may not exist. To this end, in the recently completed manuscript [30], a new integral invariance principle is derived, starting from properties of passive systems, for general nonlinear systems, generalizing the invariance principle of LaSalle and especially suitable for nonlinear control systems with observation functions. The integral invariance principle results in a broad spectrum of stabilization results which interrelate stability, observability and the converse theorems of Lyapunov, but also applies when Lyapunov functions are not known to exist. In addition to providing a criterion for attractiveness of a set, we also expect to be able to use the integral invariance principle to obtain stability results for minimal state space realizations of nonlinear systems described by I/O operators, for optimal control problems for which the value function is not smooth or even continuous, as well as for certain optimization problems with nonconvex Lagrangians which arise in the analysis of passive systems or in H^∞ design, as discussed below.

Our recent research in the development of methodologies for control system design for lumped nonlinear systems has also concentrated on the development of a systematic theory of robust control and a better understanding of underlying mechanisms for taking advantage of nonlinear effects to shape the response of complex systems. As a starting point, we recall that one of the classical problems which involves shaping the response of a system is output regulation, in which the objective is that of controlling a plant in order to have its output track (or reject) exogenous commands or disturbances. An extension of the geometric nonlinear design methodology discussed above to the important problems of asymptotic tracking and disturbance rejection has been treated by the principal investigators in a series of journal papers, one of which was awarded the 1991 George Axelby Best Paper Award by the IEEE. The nonlinear regulator theory developed in [7] is valid in case a model for an "exosystem" which generates the exogenous signal to be tracked and the exogenous disturbance to be rejected is known. The "exosystem" is also assumed to be neutrally stable, which includes many signal generators of practical interest (e.g., set-point control, oscillators and almost periodic motion) but does not allow for certain signal generators which also have application, such as tracking a ramp or a limit cycle. Even for linear systems, still more challenging cases arise either:

> when a signal generator is known but certain parameters in the system or in the disturbance channel are unknown (*structured uncertainty*); or

> when a signal generator for disturbances is not known (*unstructured uncertainty*).

Both of these problems are central focal points in the modern approach to robust control. Despite significant effort, for lumped, linear systems robust control for structured uncertainties is not nearly as well understood as robust control for unstructured uncertainties. Nonetheless, the design of control systems which are robust against structured uncertainties is of clear importance to many DOD objectives, such as the development of flight controllers for high angle-of-attack or highly agile aircraft which are robust against nonlinear variations in the coefficient of lift or nonlinear couplings between wind axis moments and rectilinear moments. In general, one can always augment the system dynamics by adding as new state variables the parameters accounting for the structured uncertainty. Of course, for linear systems this results in a nonlinear system, so that linear regulator theory cannot be directly applied and one might not expect a linear controller to exist. However, even for nonlinear systems there arises a technical problem in the application of nonlinear regulator theory as it is presently formulated; viz., the parameter values are not necessarily observable or detectable through error measurements. A new formulation of our approach to the nonlinear regulator problem, incorporating the notion of system immersion, leads to a new version of the concept of internal model and no longer requires observability or detectability. Our preliminary calculations suggest that this new formulation is also valid for problems involving unknown parameters, enabling the derivation of conditions characterizing nonlinear solvability to various local and semiglobal problems for output regulation which are robust with respect to structured perturbations. This development is very promising and we expect to be able say quite a bit more about robust control for structured uncertainties in the future.

The problem of robust regulation with respect to unstructured uncertainties is the nonlinear equivalent of the so-called *H_∞-optimal control problem* of linear system theory (see [8] [9]), in which the task of the regulator is to minimize the maximal amplitude of the frequency response of the system, when driven by disturbance inputs. The solution of the H_∞ (sub)optimal control problem via state-space methods was developed by several authors over the past few years. In the state-space formulation, the problem of minimizing the H_∞ norm (or, equivalently, the L_2 gain) of a closed loop system can be viewed as a two-person, zero sum, differential game and, thus, the existence of the desired controller can be related to the existence of a solution of the algebraic Riccati equation arising in linear quadratic differential game theory (see, e.g. [10], [11] and [12]). The advantage of the state-space approach and of the related game-theoretic interpretation is that the analysis of the H_∞ control problem for a linear system (namely, the characterization of necessary and sufficient conditions for the existence of a solution, the construction of one particular solution and the description of the set of all possible solutions) can be rendered exclusively dependent on *time domain* arguments, thus laying the basis for appropriate extensions to broader classes of systems. This point of view was suggested in [13] and in [12].

In recent years, the research efforts towards the development of a "nonlinear equivalent" of the H_∞ linear control theory have gained momentum (see [14]-[19]). Setting the problem of the attenuation of exogenous disturbances as a nonlinear differential game, the Riccati equation of the linear H_∞ control is replaced by a particular type of Hamilton-Jacobi equation, known as *Isaacs equation.* As a matter of fact, it has been rather straightforward to realize that the existence of a solution of the appropriate Isaacs equation is a sufficient condition for the existence of a *full-information* feedback law providing disturbance attenuation, in the sense of the L_2 gain [14], [17]. In addition, it has also been shown that, under appropriate assumptions, the existence of such a solution is a necessary condition for the solvability of the problem [16] and that the design of a control law based on *measurement* feedback is also possible [17]. This research effort in nonlinear disturbance attenuation is currently being developed jointly with researchers at McDonnell-Douglas Aircraft in the design of a missile autopilot for the endgame of a low visibility missile which uses a blend of reaction jet and aerodynamical surface controls.

The state space approach to the problem of disturbance attenuation naturally entails the consideration of a number of basic research issues, which have been only partially addressed so far. One of these is the existence of a state-space characterization of the L_2 gain of a system. Illustrated in [1], [20], [24], for the case of linear systems and originally in [21], [22], [23] and more recently in [16] for the more general case of nonlinear systems affine in the input, this characterization essentially provides the arguments to understand that the existence of solutions to certain Riccati, or Hamilton-Jacobi-Isaacs, equations is a necessary condition for the solvability of the problem under consideration. However, the results available in the domain of nonlinear systems are far from being a complete analog of the clear and sharp picture which is available for linear systems.

Our research [34, 35, 36] into the development of a nonlinear enhancement of H_∞ Control is based on a combination of the theory of differential games and the theory of passivity and dissipation for nonlinear systems as further developed in [6]. The techniques in [34, 35, 36] also use methods from nonlinear dynamics to ensure solvability of the steady state Hamilton-Jacobi-Isaacs equation. An analogous treatment for the finite time horizon disturbance attenuation problem is currently under development. Our preliminary analysis shows that solvability of the finite time Hamilton-Jacobi-Isaacs equation can also be approached using the geometric theory of Riccati partial differential equations, pioneered in [38, 39, 40]. In particular, [40] develops a rather complete geometric theory of classical, weak and generalized solutions to the Riccati partial differential equations arising in optimal control problems of the Bolza and Lagrange type. [41] announces necessary and sufficient conditions for the global solvability, i.e., the absence of shock waves studied in [42], for Hamilton-Jacobi-Bellman equations and for Riccati partial differential equations. Details of the proofs are provided in [43], together with the proof of a sufficient condition for the global solvability of optimal control problems by optimal controls in feedback form. This condition, which is stated in terms of convexity and concavity properties of the associated Hamiltonian function, gives a complete nonlinear enhancement of classical linear quadratic theory. The combination of geometric techniques with methods from nonlinear dynamics can, however, also apply to problems with a non-convex Lagrangian. Indeed, [44] uses these methods to give the first rigorous proof of the Kalman-Yacubovitch-Popov Lemma for a class of passive nonlinear systems.

Ultimately, the implementation of feedback control algorithms will be done in a discrete-time setting for discrete-time (sample and hold) systems, the analysis of which does not follow immediately from the continuous time theory. References [45, 46, 47, 48, 49] represent our initial efforts to extend this design philosophy to include feedback design and stabilization of discrete-time nonlinear control systems. In [48], we also establish a necessary condition for feedback stabilization of discrete-time systems, similar to the well-known condition of Brockett in continuous time. In [50, 51], we develop the theory of output regulation and robust disturbance attenuation, respectively, for discrete-time systems by a systematic use of discrete-time center and stable manifolds. A comprehensive treatment of feedback design for nonlinear discrete-time systems is the subject of the D.Sc. thesis by W. Lin, completed during the second year of this proposed research effort [26].

Of considerable practical interest is the question of the effect of boundary control in controlling or influencing the steady-state response of forced nonlinear distributed parameter systems, such as Navier-Stokes equations, which contain both nonlinear convective terms and diffusive terms (see, for example [69]-[79]). Accordingly, we have developed design methodologies in the context of a boundary control problem for the Burgers' equation on a finite interval. This remarkable system (see e.g. [80]) has been studied for some time ([81]) and was extensively developed by Burgers ([82]) as a simplified fluid flow model which nonetheless exhibits some of the important aspects of turbulence. It was later derived by Lighthill ([83]) as a second order approximation to the one dimensional unsteady Navier-Stokes equation. For Dirichlet boundary conditions, the open-loop, unforced Burgers' equation is, in fact, exponentially stable on H^1 and the series of papers [70], [71] use optimal control methods to develop feedback strategies which enhance stability on this dense subspace of L^2. In our approach, we have obtained, in a systematic way, feedback laws which asymptotically stabilize Burgers' equation with Neumann boundary conditions and for sufficiently small initial data in L^2; a case for which the open loop system is easily shown to be unstable. It is well- known [84] that the one-dimensional Burgers' equation with an external forcing term can be reduced by the Hopf-Cole substitution to the one-dimensional heat equation with a "potential" term, which might seem to offer a straightforward method for asymptotic analysis. However, for Neumann or radiation boundary conditions, this substitution introduces quadratic nonlinearities at the boundary of the interval, in sharp contrast with the case of Dirichlet boundary conditions. Rather our analysis uses modern methods of attractor theory for nonlinear partial differential evolution equations and more closely follows the classical approach developed in [77, 79] and in the work [78], in which the existence of the global attractor for two-dimensional Navier-Stokes equations was first discovered. Nonetheless, a complicating feature, which arises for boundary conditions which are neither Dirichlet nor periodic, is a nonlinear term which persists in the energy balance relation and presents considerable difficulty in obtaining a priori estimates from which the existence of an absorbing ball and the convergence of Galerkin approximations can be established as in [77, 85].

One of our longer term goals has been the development of systematic feedback design methodology for nonlinear distributed parameter systems which retain some of the intuitive appeal of classical automatic control in a spirit similar to the program pursued for the control of lumped nonlinear systems. In this direction, we have recently found significant success in designing stabilizing boundary feedback control laws for a controlled Burgers' equation on the basis of a nonlinear enhancement of root-locus techniques [86], [88]. More explicitly, in our examples, we compute the zero dynamics for the system and show that these dynamics are exponentially stable. In the language of lumped nonlinear control theory, one would say that the nonlinear system is minimum phase. Following classical design methods, in [87, 88] we implemented a proportional error feedback law, which we showed exponentially stabilized the system for small initial data in L^2. We have also been able to show that a nonlinear distributed parameter enhancement of root-locus design still persists [89, 90]: closed-loop trajectories tend to a trajectory of the open-loop zero dynamics as the gain parameters are increased to infinity. Indeed, for the controlled Burgers' equation we can prove in the high gain limit that, for sufficiently small initial data in $L^2(0,1)$, the closed loop trajectories converge in $L^\infty(0,1)$, uniformly in $t \in [0,\infty)$ to the trajectories of the corresponding zero dynamics. The extent to which this is a general phenomenon will form the nucleus of continued efforts in this area of research. In particular, we expect to be able to develop systematic feedback design methodologies for the stabilization of minimum phase nonlinear distributed parameter systems. In this regard, we are encouraged by our successful development of this methodology for the linear case, which is based upon the concepts of root locus, "instantaneous or dc gain," and zero dynamics, as well as, a feedback design methodology based on asymptotic properties of the zero dynamics. Based on this development and similar success for lumped nonlinear systems, we believe it is possible, for more general nonlinear distributed parameter control problems with unbounded inputs and outputs and a wide class of (boundary) feedback laws, to determine convergence properties of the closed loop trajectories to trajectories of the zero dynamics.

The underlying analysis for linear distributed parameter systems reposes on two new ingredients which were formulated during the first two years years of this research effort. The first is based on the discovery of a system high-frequency gain which is related to our earlier discovery of a system instantaneous gain. The high frequency gain can be obtained in terms of an appropriate asymptotic expansion of the transfer function in a right half plane, giving rise, in fact, to a fractional relative degree expressible as the difference of the orders of the input and output operators divided by the order of the spatial operator. And, in analogy with the finite dimensional case the instantaneous gain can be computed by an appropriate limit as t goes to zero in the impulse response function. We note that in the present case the impulse response function can be shown to be an L^1 function but it is not bounded at $t = 0$. Nevertheless, by multiplying by an appropriate fractional power of t, corresponding to the system relative degree, we obtain the same value for the high frequency gain and the instantaneous gain. It is worth noting that the sign of the instantaneous gain is all that is needed to determine the asymptotic behavior of the root-locus plot. To this end, fairly simple formulae have been developed for the instantaneous gain, formulae which apply to the cases considered in [91] but are also valid for a larger class of control problems including the case of noncolocated actuators and sensors. One special feature of these formulae is that they depend only on the order and coefficients of the highest order terms in the input and output boundary operators rather than the more complicated determinant condition first announced in [92]. These results have been extended in several important directions in the recent Ph.D. thesis of Jianqui He [25].

Much of the stabilization techniques reported above for nonlinear distributed parameter systems has been local, in the neighborhood of an equilibrium. For nonlinear distributed parameter systems, even global existence of solutions is often an open question, even for classically studied systems. For the Burgers' system we have studied, the zero dynamics is globally asymptotically stable, so that intuition gleaned from the lumped nonlinear theory would suggest that appropriate feedback laws might at least generate globally defined trajectories which tend to some kind of attractor. Indeed, our preliminary calculations indicate that, for the unforced Burgers' system there is a simple boundary feedback control law which, for large values of the gains, renders the single equilibrium of the closed-loop system globally Lyapunov stable, and which produces a compact attractor inside each closed ball.

Finally, we also expect to be able to continue to make progress on the problem of output regulation for this class of control systems and for a linear exogenous system. In the lumped nonlinear case, the development of a theory of steady state response relied quite heavily on center manifold theory, which is still available in infinite dimensions and would apply whenever the exogenous system is finite dimensional. For these reasons, we expect to be able to make significant progress on our third explicit research task, which is devoted to the development of a theory of output regulation for distributed parameter systems and its application to problems of asymptotic tracking and disturbance attenuation. In this connection, we are also encouraged by our preliminary calculations on global existence of solutions to the forced Burgers' system, indicating that a regulator theory for nonlinear distributed parameter systems forced by disturbances might, in principle, be developed.

4 References

References

[1] R.W. Brockett, *Finite dimensional linear systems*, Wiley, 1970.

[2] R.A. Adomaitas, "Analysis of turbocompressor flow instabilities," 1994 SIAM Annual Meeting, San Diego, CA, (July 1994), to appear.

[3] R.A. Adomaitas, and E.H. Abed, "Local nonlinear control of stall inception in axial flow compressors," *AIAA Paper*, 93-2230 (1993).

[4] R.A. Adomaitas, and E.H. Abed, "Local nonlinear control of stall in axial flow compressors," *AIAA J. Propulsion and Power*, (1994), to appear.

[5] C.I. Byrnes and A. Isidori, "Asymptotic Stabilization of Minimum Phase Nonlinear Systems," *IEEE Trans. Aut. Control*, 36 (1991) 1122-1137.

[6] C.I. Byrnes, A. Isidori and J.C. Willems, "Passivity, Feedback Equivalence and the Global Stabilization of Minimum Phase Nonlinear Systems," *IEEE Trans. Aut. Control*, 36 (1991) 1228-1240.

[7] A.Isidori, C.I.Byrnes, "Output regulation of nonlinear systems, " *IEEE Trans. Autom. Control*, *AC-35*: 131–140, 1990.

[8] G. Zames, "Feedback and optimal sensitivity: model reference transformations, multiplicative seminorms and approximate inverses," *IEEE Trans. Autom. Control*, *AC-26*:301–320, 1981.

[9] B.A.Francis, *A Course in H_∞ Control Theory*, Springer Verlag, 1987.

[10] J.C. Doyle, K. Glover, P.P. Khargonekar, and B.A. Francis, "State space solutions to standard H_2 and H_∞ control problems," *IEEE Trans. Autom. Control*, *AC-34*:831–846, 1989.

[11] G. Tadmor, "Worst case design in time domain," *Math. Control, Signals and Systems*, *3*:301–324, 1990.

[12] T. Basar and P. Bernhard, *H_∞-optimal control and related Minimax problems*, Birkhauser, 1990.

[13] J.A. Ball and J.W. Helton, "H_∞ control for nonlinear plants: connection with differential games," In *Proc. of 28th Conf. Decision and Control*, pages 956–962, Tampa, FL, December 1989.

[14] A.J. Van der Schaft, "A state-space approach to nonlinear H_∞ control," *Syst. and Contr. Lett.*, *16*:1–8, 1991.

[15] A.J. Van der Schaft, " Complements to nonlinear H_∞ optimal control by state feedback," *IMA J. Math. Contr. Inf.*, *9*:245–254, 1992.

[16] A.J. Van der Schaft, "L_2-gain analysis of nonlinear systems and nonlinear H_∞ control," Tech. Memorandum 969, Universiteit Twente, May 1991.

[17] A. Isidori and A. Astolfi, "Nonlinear H_∞ control via measurement feedback," *J. Math. Systems, Estimation and Control*, *2*:31–44, 1992.

[18] A. Isidori and A. Astolfi, "Disturbance attenuation and H_∞ control via measurement feedback in nonlinear systems, " *IEEE Trans. Autom. Control*, *AC-37*:1283–1293, 1992.

[19] A. Isidori and A. Astolfi, "Nonlinear H_∞ control via measurement feedback for affine nonlinear systems, " *Int J. Robust and Nonlinear Control*, 4 (1994), pp. 553-574.

[20] B.D. Anderson, "An algebraic solution to the spectral factorization problem," *IEEE Trans. Autom. Control*, *AC-12*:410–414, 1967.

[21] P.J. Moylan, "Implications of passivity in a class of nonlinear systems," *IEEE Trans. Autom. Control*, *AC-19*:373–381, 1974.

[22] D. Hill and P.J. Moylan, "The stability of nonlinear dissipative systems," *IEEE Trans. Autom. Control*, *AC-21*:708–711, 1976.

[23] J.C. Willems, "Dissipative dynamical systems Parts I: systems with quadratic supply rates," *Arch. Rat. Mech. Anal.*, Vol. 45, (1972), 321-351, (1972).

[24] J.C. Willems, "Least square optimal control and the algebraic Riccati equation," *IEEE Trans. Autom. Control*, *AC-16*:621–634, 1971.

[25] J. He, "A root locus design methodology for parabolic distributed parameter systems," Ph.D. Thesis at Texas Tech University, (1993).

[26] W. Lin, "Feedback design for nonlinear discrete-time control systems," Ph.D. Thesis at Washington University, St.Louis, (1993).

[27] C.I. Byrnes and A. Isidori, "On the Attitude Stabilization of Rigid Spacecraft," *Automatica*, 27 (1991) 87-95.

[28] C.I. Byrnes, A. Isidori, S. Sastry, and P. Kokotovic, "Singularly Perturbed Zero Dynamics of Nonlinear Systems," *IEEE Trans. Aut. Control*, 36, 1991.

[29] C.I. Byrnes and J. Roltgen, "G-Field control of nonlinear systems," *Proceedings of AIAA Conference on Guidance and Control*, Hilton Head, (1992).

[30] C.I.Byrnes and C.F. Martin, "An Integral Invariance Principle for Nonlinear Systems," to appear in *IEEE Trans. Aut. Control.*

[31] C.I. Byrnes and S.V. Pandian, "Exponential observer design," to appearin *Proceedings of Second IFAC NOLCOS*, Bordeaux, 1992.

[32] C.I. Byrnes and A. Isidori, "Asymptotic Tracking and Disturbance Rejection in Nonlinear Systems *New Trends in Systems Theory*, Birkhauser-Boston, 1991.

[33] C.I. Byrnes and A. Isidori, "Asymptotic Tracking and Disturbance Rejection in Nonlinear Systems *New Trends in Systems Theory*, Birkhäuser-Boston, 1991.

[34] F. Delli Priscoli, A. Isidori, "Robust tracking for a class of nonlinear systems," *Proc. 1st European Control Conf.* Grenoble, (1991).

[35] A. Isidori, "Feedback control of nonlinear systems," *Proc. 1st European Control Conf.*, Grenoble, (1991).

[36] A. Isidori, "Disturbance attenuation for nonlinear systems," to appear in *Math. Systems, Estimation, and Control.*

[37] A. Isidori, "A Necessary Condition for Nonlinear H_∞ Control via Measurement Feedback," *Systems and Control Lett.*, 23 (1994), pp. 169-177.

[38] C.I. Byrnes, "New methods for nonlinear optimal control," *Proceedings of First European Control Conf.*, Grenoble 1991.

[39] C.I. Byrnes, "Some partial differential equations arising in nonlinear control and optimization," *Computation and Control, II*, Birkhäuser-Boston, (1991).

[40] C.I. Byrnes, "Riccati partial differential equations for nonlinear Lagrange and Bolza problems," submitted to *Math. Systems, Estimation, and Control.*

[41] C.I. Byrnes and H. Frankowska, "Unicite des controles optimales et chocs pour les equationes Hamilton-Jacobi-Bellman et Riccati,"submitted to Comptes Rendus Acad. Sci. Paris.

[42] C.I.Byrnes and A. Jhemi, "Shock Waves for Riccati Partial Differential Equations Arising in Nonlinear Optimal Control," *Systems, Models and Feedback: Theory and Applications* (A. Isidori, T.J. Tarn, eds.) Birkhäuser-Boston, 1992.

[43] C.I. Byrnes and H. Frankowska, "Uniqueness of optimal controls and the nonexistence of shocks for Hamilton-Jacobi-Bellman and Riccati partial differential equations," submitted to *SIAM Journ. Control and Opt.*

[44] C.I.Byrnes and K. Doll, "On the Kalman-Yacubovitch-Popov Lemma for Nonlinear Systems," *Computation and Control III*, Birkhäuser-Boston, 1993.

[45] C.I. Byrnes and W. Lin, "Feedback stabilization of discrete time nonlinear systems," to appear in *Systems and Control Letters.*

[46] C.I. Byrnes and W. Lin, "Lossless nonlinear systems, stability and feedback equivalence in discrete time," in preparation.

[47] C.I. Byrnes and W. Lin, "Discrete-time lossless systems, feedback equivalence and passivity," *32nd IEEE Conf. Dec. and Control*, San Antonio, TX, 2, 1775-1781, December, 1993.

[48] C.I. Byrnes and W. Lin, "On Discrete-Time Nonlinear Control," *32nd IEEE Conf. Dec. and Control*, San Antonio, TX, 2, 2990-2996, December, 1993.

[49] C.I. Byrnes and W. Lin, "Passivity and Absolute Stability for a Class of Discrete-time Nonlinear Control Systems," *Automatica*, Vol.31, No.3, March, 1995.

[50] C.I. Byrnes and W. Lin, "Design of discrete time nonlinear control systems via smooth feedback," to appear in *IEEE Trans. Aut. Cont.*

[51] W. Lin and C.I. Byrnes, "H^∞ control of discrete time nonlinear systems via state and full information feedback," submitted to *IEEE Trans. Aut. Cont.*

[52] A. Isidori, "New Results on Nonlinear H_∞ Control via Measurement Feedback," *Advances in Dynamic Games and Applications*, Birkhauser-Boston. 56-69, 1994.

[53] A. Isidori, and T.J. Tarn, "Dissipation Inequalities for a Class of Composite Systems, with Applications to Constrained Disturbance Attenuation" *Proc. of 32nd IEEE Conf. Decision and Control*, 2, 1542-1547, December 1993.

[54] S. Suzuki, A. Isidori and T.J. Tarn, "H_∞ Control of Nonlinear Systems with Sampled Measurements," *1st Asian Control Conference*, 2, 41-44, July, 1994.

[55] C.I. Byrnes and W. Lin, "Losslessness, Feedback Equivalence and the Global Stabilization of Discrete-time Nonlinear Systems," *IEEE Transactions Automatic Control*, 39, 83-98, January, 1994.

[56] W. Lin and C.I. Byrnes , "Design of Discrete-time Nonlinear Control Systems via Smooth Feedback," *IEEE Transactions Automatic Control*, Vol.39,No.11, November, 1994.

[57] W. Lin, "Global Stabilization of Nonlinear Systems via State and Output Feedback," *IEEE Transactions Automatic Control*, Vol.40, No.7, July, 1995.

[58] W. Lin and C.I. Byrnes, 'Remark on Linearization of Discrete-time Autonomous Systems and Nonlinear Observer Design," *Systems and Control Letters* Elsevier, NH, 23, December, 1994.

[59] W. Lin, "Feedback Stabilization of General Nonlinear Control Systems: A Passive Systems Approach," *Systems and Control Letters* Elsevier, NH, 23, December, 1994.

[60] W. Lin and C.I. Byrnes, "Zero-State Observability and Stability of Discrete-time Nonlinear Systems," *Automatica* Vol.31, No.3, March, 1995.

[61] W. Lin and C.I. Byrnes, "Discrete-time Nonlinear H_∞ Control with Measurement Feedback," *Automatica* Vol.31, No.4, April, 1995.

[62] W. Lin and C.I. Byrnes, "KYP Lemma, State Feedback and Dynamic Output Feedback in Discrete-time Bilinear Systems," *Systems and Control Letters* Elsevier, NH. 23, 127-136, August, 1994.

[63] W. Lin, and C.I. Byrnes, "Dissipativity, L_2-gain and H_∞ Control for Discrete-Time Nonlinear Systems," *Proceedings of 1994 America Control Conference* 2257-2260, July, 1994.

[64] W. Kang, and C.I. Byrnes, "Stability, detectability and the problem of disturbance decoupling," *Systems and Networks: Mathematical Theory and Applications*, Vol. 1 Akademie Verlag, Berlin, 77, 71-84, September, 1994.

[65] A. Isidori and W.Kang, "H_∞ Control via Measurement Feedback for General Nonlinear Systems," *IEEE Trans. on Automatic Control*, scheduled to appear in the April 1995 issue.

[66] W. Kang, "Approximate Linearization of Nonlinear Control Systems," *Systems and Control Letters* 23, No. 1, 43-52, 1994.

[67] "Nonlinear H_∞ control and its application to rigid spacecraft," to appear in *IEEE Trans. Automat. Contr.*.

[68] "Extended controller form and invariants of nonlinear control systems with a single input," *J. of Mathematical System, Estimation and Control*, vol. 4, No. 2, 1994, pp253-256.

[69] A.V. Babin, I.M. Vishik "Attractors of partial differential equations in an unbounded domain," *Proceedings of the Royal Society of Edinburgh*, 116A (1990) 221-243.

[70] J.A. Burns and S. Kang, "A control problem for Burgers' equation with bounded input/output," *Nonlinear Dynamics*, (1991), Vol. 2, 235-262.

[71] J.A. Burns and S. Kang, "A Stabilization problem for Burgers' equation with unbounded control and observation," *International Series of Numerical Mathematics*, (1991), Vol. 100, 51-72.

[72] C.I. Byrnes, D.S. Gilliam, "Boundary feedback design for nonlinear distributed parameter systems," *Proc. of IEEE Conf. on Dec. and Control*, Britton, England 1991.

[73] D.S. Gilliam and C.I. Byrnes, "Stability of certain distributed parameter systems by low dimensional controllers: a root locus approach," *Proceedings 29th IEEE International Conference on Decision and Control.*

[74] Constantin, P. Foias, C. Temam, R. "Attractors representing turbulent flows," *Memoirs of A.M.S.* 53 (1985) No. 314.

[75] C. Foias, R. Temam, "Some analytic and geometric properties of the solutions of the Navier-Stokes equations," *J. Math. Pures Appl.* 58 (1979) 339-368.

[76] Shubov, V. "Long time behavior of infinite dimensional dissipative dynamical systems," *Journal of Math. Systems, Estimation and Control*, 2 (1992), 381-427.

[77] Ladyzhenskaya, O.A. *The Mathematical Theory of Viscous Incompressible flow*, 2nd ed. Gordon and Breach, New York, 1969.

[78] Ladyzhenskaya, O.A. "The dynamical system that is generated by the Navier-Stokes equations," *Journal of Soviet Mathematics*, 3 (1975) 458-479.

[79] Ladyzhenskaya, O.A., Solonnikov, V.A., Ural'ceva, N.N. *Linear and Quasilinear Equations of Parabolic Type*, Translations of AMS, Vol. 23, 1968.

[80] C.A.J. Fletcher, "Burgers' Equation: A Model for all Reasons," *Numerical Solutions of Partial Differential Equations*, J. Noye (Editor), North-Holland Publ. Co. (1982).

[81] H. Bateman, "Some recent researcheson the motion of fluids," *Mon. Weather Rev.*Vol. 43, 163-170, (1915).

[82] J. Burgers, "Application of a model system to illustrate some points of the statistical theory of free turbulence," *Nederl. Akad. Wefensh. Proc.*, 43 (1940), 2-12.

[83] M.J. Lighthill, "Viscousity effects in sound waves of finite amplitutde," *Surveys in Mechanics*, (eds. G.K. Batchlor and R.M. Davies), C.U.P., Cambridge, 250-351, (1956).

[84] E. Hopf, "The partial differential equation $u_t + uu_x = \mu u_{xx}$," *Comm. Pure Appl. Math.*, 3 (1950), 201-230.

[85] Temam, R. *Navier-Stokes Equations, Theory and Numerical Analysis*, 3rd rev. ed., North-Holland, Amsterdam, 1984.

[86] C.I. Byrnes, D.S. Gilliam, "Boundary Feedback Stabilization of a Controlled Viscous Burgers' Equation," *Proc. of IEEE Conf. on Dec. and Control*, Tucson, AZ, 1992.

[87] C.I. Byrnes, D.S. Gilliam and V.I. Shubov, "Boundary control, for a viscous burgers' equation," *Identification and control in systems governed by partial differential equations* H.T. Banks, R.H. Fabiano and K. Ito (Eds.), SIAM, 1993, 171-185.

[88] C.I. Byrnes, D.S. Gilliam and V.I. Shubov, "Boundary control, feedback stabilization and the existence of attractors for a viscous burgers' equation," submitted to *SIAM J. Cont. Opt.*.

[89] C.I. Byrnes, D.S. Gilliam and V.I. Shubov, "High gain limits of trajectories and attractors for a boundary controlled viscous Burgers' equation," to appear in *J. Math. Sys., Est. and Control*

[90] C.I. Byrnes, D.S. Gilliam and V.I. Shubov, "Convergence of trajectories for a controlled viscous Burgers' equation," to appear in *Control and Estimation of Distributed Parameter Systems: Nonlinear Phenomena* Birkhauser-Boston, 1994.

[91] C.I.Byrnes, D.S. Gilliam and J.He, "Root locus and boundary feedback design for a class of distributed parameter systems," *SIAM J. Control and Opt.* 32, No. 5, 1364-1427, September, 1994.

[92] C.I.Byrnes, D.S. Gilliam and J.He, "A Root Locus Methodology for Parabolic Distributed Parameter Feedback Systems," *Computation and Control II*, Birkhäuser-Boston, 1991, 63-83.

[93] C.I. Byrnes, D.S. Gilliam and V.I. Shubov, "Zero and pole dynamics for a controlled Burger's equation," to appear in *Proceedings IEEE CDC*, December, 1994.

[94] C.I. Byrnes, D.S. Gilliam and V.I. Shubov, "Global Lyapunov stabilization of a nonlinear distributed parameter system," to appear in *Proceedings IEEE CDC*, December, 1994.

[95] C.I. Byrnes, D.S. Gilliam and V.I. Shubov, "Zero and pole dynamics for a controlled Burger's equation," to appear in *Proceedings IEEE CDC*, December, 1994.

[96] D.S. Gilliam, D. Lee, C.F. Martin and V.I. Shubov, "Turbulent Behavior for a Boundary Controlled Burger's equation," to appear in *Proceedings IEEE CDC*, December, 1994.

[97] C.I. Byrnes, D.S. Gilliam, V.I. Shubov and Z. Xu, "The effect of viscousity on the steady state response of a nonlinear system," to appear in *Proceedings Fourth Conference on Computation and Control* Bozeman, Montana, August, 1994.

[98] H.T. Banks, D.S. Gilliam, and V.I. Shubov, "Well-posedness for a one dimensional nonlinear beam," to appear in *Proceedings Fourth Conference on Computation and Control* Bozeman, Montana, August, 1994.

5 Participating Professionals

1. Principal Investigators
   - Christopher I. Byrnes
   - Alberto Isidori
2. Postdocs
   - David S. Gilliam

- Wei Kang
- Wei Lin

3. Graduate Students

   - D. Gupta, Ph.D., Arizona State University,

     May 1993.
     Thesis Title: Global Analysis of Splitting Subspaces

   - W. Lin, Ph.D. candidate, Washington University.

     August, 1993.
     Thesis Title: Stabilization and Control of Discrete-time Nonlinear Systems

   - J. He, Ph.D. Texas Tech University, June 1993

     May, 1993.
     Thesis Title: Root Locus for Birkhoff Regular Boundary Control Systems

   - J. Roltgen, Ph.D. candidate, Washington University.

     anticipated graduation date, May, 1994.
     Thesis Title: Inner-loop, Outer-loop Stabilization of Nonlinear Systems

   - K. Doll, D.Sc. Washington University, anticipated 1994

     anticipated graduation date, May, 1994.
     Thesis Title: Robust Stabilizatioin of Nonlinear Systems

   - S.V. Pandian, D.Sc. Washington University,

     anticipated graduation date, May, 1994.
     Thesis Title: Nonlinear Observer Design

   - R. Eberhardt, D.Sc. Washington University,

     anticipated graduation date, May, 1994.
     Thesis Title: Nonlinear Optimal Control with Input Constraints

6 Scientific Interactions

In addition to collaborative research with engineering research and development personnel at McDonnell-Douglas Aircraft Co., St. Louis, MO, and scientific interaction with AFOSR personnel at Bolling AFB, Ft. Eglin AFB and Wright Patterson AFB, we have presented many inivited lectures and colloquia nationally and internationally:

August 1991

"Partial Differential Equations and Nonlinear Control," Invited lecture presented by Dr. Christopher I. Byrnes at the AFOSR-Washington University Conference on Nonlinear Control and Its Applications, St. Louis.

"Disturbance attenuation in nonlinear systems," Invited lecture presented by Dr. Alberto Isidori at the AFOSR-Washington University Conference on Nonlinear Control and Its Applications, St. Louis.

October 1991

"Tikhonov Regularization for inverse boundary control problems," Invited lecture presented by Dr. David Gilliam at the South West Analysis Conference, Lafayette, LA.

November 1991

"H_∞ control for Nonlinear Systems," Invited lecture presented by Dr. Alberto Isidori at the University of California, Berkeley, California.

"H_∞ control for Nonlinear Systems," Invited lecture presented by Dr. Alberto Isidori at the California Institute of Technology, Pasadena, California.

December 1991

"Boundary feedback stabilization for nonlinear distributed parameter systems," Invited lecture presented by Dr. David Gilliam at the 30th IEEE Conference on Decision and Control, Brighton, England,

January 1992

"Geometric Methods for Nonlinear Optimal Control," Invited colloquium presented by Dr. Christopher I. Byrnes at Tokyo University, Japan.

Febuary 1992

"Boundary feedback control of a viscous Burgers' equation," Invited lecture presented by Dr. David Gilliam at the Department of Systems Science and Mathematics, Washington University, St. Louis.

March 1992

"Root locus and boundary feedback design for distributed parameter systems," Invited lecture presented by Dr. David Gilliam at the Department of Systems Science and Mathematics, Washington University, St. Louis.

"Geometric Methods for Nonlinear Optimal Control," Invited lecture presented by Dr. Christopher I. Byrnes at Conference on Nonlinear Systems and Control, Centre de Physique, Les Houches, France.

"H_∞ control for Nonlinear Systems," Invited lecture presented by Dr. Alberto Isidori at the Workshop on Control Theory, Oberwolfach, Germany.

April 1992

"Boundary feedback stabilization of a controlled Burgers' system," Invited lecture presented by Dr. David Gilliam at the Institute for Computational and Applied Mathematics, Virginia Polytech and State University.

"A Root locus design methodology for distributed parameter systems," Invited lecture presented by Dr. David Gilliam at the Department of Electrical Engineering, University of Maryland.

May 1992

"Geometric Methods for Nonlinear Control," Invited lecture presented by Dr. Christopher I. Byrnes at Eglin Air Force Base Guidance and Control Conference, Eglin AFB, Florida.

"Shock Waves for Riccati Equations Arising in Nonlinear Optimal Control," Invited lecture presented by Dr. Christopher I. Byrnes at the NSF-Washington University Conference on Nonlinear Systems

"Disturbance attenuation in nonlinear systems," Invited lecture presented by Dr. Alberto Isidori at the NSF-Washington University Conference on Nonlinear Systems.

June 1992

"Convergence of trajectories of a controlled Burgers' system," Invited lecture presented by Dr. David Gilliam at the 10th Annual Joint Summer Research Conference in Mathematical Sciences: Control and Identification of Partial Differential Equations, Mount Holyoke College, South Hadley, Mass.

"Shock Waves for Riccati Equations Arising in Nonlinear Optimal Control," Invited lecture presented by Dr. Christopher I. Byrnes at the Conference on Systems and Feedback, Capri, Italy.

"Attenuation of disturbances in nonlinear systems," Invited lecture presented by Dr. Alberto Isidori at the Conference on Systems and Feedback, Capri, Italy.

July 1992

"Exponential Observers for Nonlinear Systems," Invited lecture presented by Dr. Christopher I. Byrnes at the IFAC Conference on Nonliear Control, Bordeaux, France.

"Approximate model matching of nonlinear systems," Invited lecture presented by Dr. Alberto Isidori at the IFAC Conference on Nonliear Control, Bordeaux, France.

August 1992

"Boundary stabilization and attractors for a controlled Burgers' equation," Plenary lecture presented by Dr. David Gilliam at the Third Conference on Computation and Control,

Bozeman, Montana.

"Nonlinear Optimal Control for Nonconvex Lagrangians," Plenary lecture presented by Dr. Christopher I. Byrnes at the Third Conference on Computation and Control, Bozeman, Montana.

September 1992

"Convergence of trajectories of a controlled Burgers' system," Invited lecture presented by Dr. David Gilliam at the SIAM Conference on Control, Minneapolis.

"Shock Waves for Riccati Equations Arising in Nonlinear Optimal Control," Invited lecture presented by Dr. Wei Kang at the SIAM Conference on Control, Minneapolis.

October 1992

"Shock Waves for Riccati Equations Arising in Nonlinear Optimal Control," Invited lecture presented by Dr. Christopher I. Byrnes at SIAM Conference on Nonlinear Dynamical Systems, Snowbird, Utah.

Invited lecture "Boundary control of a viscous Burgers' equation," at North Carolina State University, lecture presented by Professor David S. Gilliam.

"Existence and absence of shock waves for Riccati and Hamilton-Jacobi-Bellman equations for optimal control" Invited lecture at SIAM Conference on Nonlinear Dynamics and PDE's," lecture presented by Professor Christopher I. Byrnes.

November 1992

"Boundary control of a viscous Burgers' equation," Invited lecture presented by Dr. David Gilliam at the Annual Students SIAM meeting at Texas Tech University.

December 1992

"Root locus for distributed parameter systems," Invited lecture presented by Dr. David Gilliam at the Department of Mathematics, University of Texas Dallas.

"L^2 existence and local attractors for Burgers' equation," Invited lecture presented by Dr. David Gilliam at the Texas Systems Days.

"Boundary feedback stabilization for a viscous Burgers' equation," Invited lecture presented by Dr. David Gilliam at the 31st Conference on Decision and Control, Tucson.

"On the nonlinear dynamics of fast filtering algorithms," at the 31st IEEE Conference on Decision and Control, Tucson. Invited paper authored by C.I. Byrnes, A. Lindquist, and Y.S. Zhou, lecture presented by Ms. Zhou.

"Root-locus and boundary feedback design for parabolic distributed parameter systems," at the 31st IEEE Conference on Decision and Control, Tucson. Invited paper authored by C.I. Byrnes, D. S. Gilliam and J. He, lecture presented by Mr. He.

January 1993

"Poincaré technique and computational nonlinear system control," Invited lecture, at the Department of Mathematical Sciences, IUPUI, presented by Professor Wei Kang.

"Stability, observability and the converse theorem of Lyapunov for nonlinear systems," Invited lecture at the Department of Optimization and Systems, Royal Institute of Technology, Stockholm, lecture presented by Professor Christopher Byrnes.

April 1993

"H_∞ Control via Measurement Feedback for Affine Nonlinear Systems," Invited lecture at the University of Augsburg (Germany), lecture presented by Professor A. Isidori.

May 1993

"Necessary Conditions for Nonlinear H_∞ Control via Measurement Feedback," Invited lecture at the University of California (Santa Barbara), lecture presented by Professor A. Isidori.

"Boundary control and attractors for a viscous Burgers' equation," Invited lecture at Conference on Nonlinear Partial Differential Equations at Snow Bird Utah, lecture presented by Professor David S. Gilliam.

"Nonlinear Control Systems," Invited lecture at AFOSR Workshop on Dynamics and Control at University of Michigan, Ann Arbor, lecture presented by Professor Christopher I. Byrnes.

June 1993

"Regularity of solutions and attractors for Burgers' equation," Invited lecture at University of Texas at Dallas, lecture presented by Professor David S. Gilliam.

"An integral invariance principle for nonlinear systems," Invited lecture at Laboratoire pour Systemes et Signaux, CNRS, Gif-sur-Yvette, lecture presented by Professor Christopher I. Byrnes.

"An integral invariance principle for nonlinear systems," Invited lecture at CNRS-Fountainbleu, lecture presented by Professor Christopher I. Byrnes.

"An integral invariance principle for nonlinear systems," Invited lecture at Dipartimento di Informatica e Sistemistica, Universita di Roma "La Sapienza", lecture presented by Professor Christopher I. Byrnes.

"An integral invariance principle for nonlinear systems," Invited lecture at Dipartimento di Matematica, Universita di Firenza, lecture presented by Professor Christopher I. Byrnes.

"An integral invariance principle for nonlinear systems," Invited lecture at Dipartimento di Elettronica e Sistemistica, Universita di Padova lecture presented by Professor Christopher I. Byrnes.

July 1993

"An integral invariance principle for nonlinear systems," Invited lecture at the International Conference on Control and Estimation of Distributed Parameter Systems: Nonlinear Phenomena, Vorau, Austria, lecture presented by Professor Christopher I. Byrnes.

"Convergence of trajectories for a controlled viscous Burgers' equation," Invited lecture at the International Conference on Control and Estimation of Distributed Parameter Systems: Nonlinear Phenomena, Vorau, Austria, lecture presented by Professor David s. Gilliam.

August 1993

"Stability, observability and the converse theorem of Lyapunov for nonlinear systems," Plenary address at the 1993 International Conference on the Mathematical Theory of Networks and Systems, Regensburg, Germany, lecture presented by Professor Christopher I. Byrnes.

September 1993

"Convergence of trajectories for a boundary controlled viscous Burgers' equation," Invited lecture presented at the University of Texas at Dallas, lecture presented by Professor David S. Gilliam.

October 1993

"An integral invariance principle for nonlinear systems," Invited lecture at the Department of Optimization and Systems, Royal Institute of Technology, Stockholm, lecture presented by Professor Christopher I. Byrnes.

"Robust Regulation for Affine Nonlinear Systems Subject to Unstructured Uncertainties", Invited lecture at Boston University, lecture presented by Professor Alberto Isidori.

"Robust Regulation for Affine Nonlinear Systems Subject to Unstructured Uncertainties", Invited lecture at Yale University, lecture presented by Professor Alberto Isidori.

November 1993

"Robust Regulation of Nonlinear Systems", Invited lecture at the University of California, Santa Barbara, lecture presented by Professor Alberto Isidori.

"The effect of boundary control on the structure of stationary solutions for Burgers' equation," Invited lecture at Annual Conference "Texas Systems Days" held at the University of Texas Arlington, lecture presented by Professor David S. Gilliam.

December 1993

"Discrete-time lossless systems, feedback equivalence and passivity," Lecture presented at 32nd IEEE Conf. Dec. and Control, San Antonio, TX (1993), lecture presented by Professor W. Lin.

"On Discrete time Nonlinear Control," Lecture presented at 32nd IEEE Conf. Dec. and Control, San Antonio, TX (1993), lecture presented by Professor W. Lin.

January 1994

"Structure of stationary solutions for a controlled viscous Burgers' equation," Invited lecture presented at the University of Texas at Dallas, lecture presented by Professor David S. Gilliam.

"Structure of stationary solutions for a controlled viscous Burgers' equation," Invited lecture presented at Washington University, St. Louis, lecture presented by Professor David S. Gilliam.

February 1994

Technical briefing at Wright Patterson AFB, Prof. C.I. Byrnes

March 1994

"A Complete Parameterization of Positve Rational Covariance Extensions," Invited lecture at Oberwolfach, lecture presented by Professor C.I. Byrnes

"Robust Regulation for Nonlinear Systems in the Presence of Unstructured Perturbations", Invited lecture at Oberwolfach, lecture presented by Professor Alberto Isidori.

"A Complete Parameterization of Positve Rational Covariance Extensions," Invited lecture at Universita di Padova, lecture presented by Professor C.I. Byrnes

"Output Regulation in Nonlinear Systems", Invited lecture at Sheffield, lecture presented by Professor Alberto Isidori.

"Nonlinear H_∞ control and its applications," Invited talk in the Department of Mathematics, Naval Postgraduate School, lecture presented by Professor W. Kang.

April 1994

- "Structure stationary solutions and attractors for a controlled viscous Burgers' equation," Invited lecture presented at Texas Partial Differential Equations Conference at the University of Texas at Austin lecture presented by Professor David S. Gilliam.

May 1994

"Nonlinear H_∞ control and its applications," Invited talk in the Department of Mathematics, California State Polytechnic University, lecture presented by Professor W. Kang.

"Feedback Stabilization About Attractors," Invited lecture at Eglin Air Force Base Guidance and Control Conference, Eglin AFB, Florida, lecture presented by Professor C.I. Byrnes

June 1994

"A Complete Parameterization of Positve Rational Covariance Extensions," Invited lecture at Royal Institute of Technology, Stockholm, lecture presented by Professor C.I. Byrnes

August 1994

"Feedback Stabilization About Attractors" Invited lecture presented at the fourth Conference on Computation and Control held at Montana State University, Bozeman, Montana, lecture presented by Professor C.I. Byrnes

"The effect of viscousity on the steady state behavior of solutions to Burgers' equation," Invited lecture presented at the fourth Conference on Computation and Control held at Montana State University, Bozeman, Montana, lecture presented by Professor David S. Gilliam.

September 1994

"Feedback Stabilization About Attractors and Inertial Manifolds," Invited lecture at Royal Institute of Technology, Stockholm, lecture presented by Professor C.I. Byrnes

7 New Discoveries

An important part of this research effort in nonlinear feedback control, documented here, is concerned with the design of nonlinear feedback laws which stabilize a given nonlinear control system, either about an equilibrium or an attractor. Interest in the development of a systematic methodology for nonequilibrium stabilization is becoming more widely appreciated as the benefits to taking advantage of dominant nonlinear effects have become more apparent. In a recent paper, which won the 1993 IFAC Automatica Best Paper Award, an enhancement of the geometric methodology developed for the equilibrium case is applied to the derivation of feedback laws to induce a stable limit cycle in the center manifold of a rigid spacecraft system which cannot be asymptotically stabilized about a reference equilibrium, even by nonlinear feedback. As an initial general result in this research effort, we have recently derived a theorem characterizing the existence of globally asymptotically stable compact attractors in terms of feedback equivalence to passive systems, for a suitable choice of system output. This result promises to have interesting corollaries extending our previous application involving feedback stabilization of limit cycles, as well as for more general attractors. Our recent research in the development of methodologies for control system design for lumped nonlinear systems has also concentrated on the development of a systematic theory of robust control, both for structured and unstructured uncertainties.

For lumped nonlinear systems, we have also made a significant new discovery, the integral invariance principle, which generalizes LaSalle's Invariance Principle. Not requiring a Lyapunov or energy function which is continuously defined on an open set of initial conditions, this principle can also apply to convergence of trajectories to attractors which are not uniformly attractive - such as arise in the Lorenz attractor.

Another important discovery is the fact that, by formulating the robust control problem in terms of a dissipation inequality, one can derive necessary and sufficient conditions for the solution of the H^∞ control problem with measurement feedback. In particular, sufficient conditions can be derived for a separation principle to hold for the problems of H^∞ state feedback control and for the problem of robust, or H^∞, estimation.

Very significant as well is the discovery that, for optimal control problems with a convex Lagrangian (or performance measure), that uniqueness of the optimal control problem is equivalent to global existence of the solution to the Hamilton-Jacobi-Bellman equation. This was discovered, under suitable hypotheses, by analyzing the onset of shocks to the associated Riccati PDE.

We have made unanticipated advances for distributed parameter systems. For linear parabolic distributed parameter systems, we have discovered the distributed parameter analogue of important frequency domain constructs such as high frequency (dc) gain and a (fractional) relative degree, expressible in terms of a fractional Laurent expansion of the system transfer function. This allows for the first rigorous development of root- locus methods for feedback design for this important class of distributed parameter systems.

For certain nonlinear distributed parameter systems, we have discovered the nonlinear enhancement of root-locus methods for feedback design. Of considerable practical interest is the question of the effect of boundary control in controlling or influencing the steady-state response of forced nonlinear distributed parameter systems, such as Navier-Stokes equations, which contain both nonlinear convective terms and diffusive terms. Accordingly, we have developed design methodologies for nonlinear distributed parameter systems in the context of a boundary control problem for the Burgers' equation on a finite interval. In this direction, we have recently found significant success in designing stabilizing boundary feedback control laws for a controlled Burgers' equation on the basis of a nonlinear enhancement of root-locus techniques. More explicitly, in our examples, we compute the zero dynamics for the system and show that these dynamics are exponentially stable. In the language of lumped nonlinear control theory, one would say that the nonlinear system is minimum phase. Following classical design methods, we have implemented a proportional error feedback law, which we showed exponentially stabilized the system for all sufficiently small, square integrable initial data. We have also been able to show that a nonlinear distributed parameter enhancement of root-locus design still persists: closed-loop trajectories tend to a trajectory of the open-loop zero dynamics as the gain parameters are increased to infinity. Finally, our preliminary calculations indicate that, for the unforced Burgers' system there is a simple boundary feedback control law which renders the equilibrium of the closed-loop system globally Lyapunov stable, and which produces a compact attractor inside each closed ball.

8 Additional Information

As described in Section 1 and the specific research tasks described in our proposal, our work on distributed parameter control is a preliminary to one of our longer range research goals, the regulator problem for linear and nonlinear distributed parameter systems. One of the motivating applications for our basic research in nonlinear control is flow control, e.g. the control of fluid flow across an airfoil using thermal actuators, and related topics such as combustion and noise control currently being researched in experimental laboratories in the United States and Japan. Here, more complicated dynamical behavior is exhibited in both the process to be controlled and the actuator. While there is a definite need, for the purpose of control, for the development of specific alternative models to the Navier Stokes equations, it is extremely likely that simplified models exhibiting some of the complicated behavior encountered in turbulent fluid flow will involve nonlinear and also infinite dimensional systems. Indeed, the recent "Fleming Report" on Future Directions in Control Theory emphasized research on flow control as an important research need for the future of American technology.

The problems of stabilizing and controlling nonlinear systems are limiting factors in the design of several DOD systems. For example, there is current research and development effort in the aerospace industry dedicated toward stabilization and control of high performance aircraft operating in nonlinear flight conditions involving agility and high angles-of-attack. Because linear systems exhibit much more predictable and well-understood behavior, the control of linear systems has been more highly developed than the control of nonlinear systems. For this reason, current approaches to flight control in the presence of nonlinear effects, e.g. "gain scheduling", have typically involved finding an "equivalent" linear system, for which a controller is then designed using existing linear methods. However, for more highly nonlinear maneuvers involving increased agility and higher angle-of-attack, the limitations of conventional design methods stem from the lack of a reasonable "equivalent" linear system which incorporates in some way the increasingly dominant nonlinear effects. This research effort in nonlinear stabilization and control is aimed at developing a systematic methodology to overcome some of these limitations. The Fleming Report also emphasized the importance of basic problems such as nonlinear feedback stabilization to the future of the American research effort in control, underscoring the earlier consensus of the 1986 IEEE Santa Clara meeting which stated that "nonlinear feedback stabilization is by far... the most important open problem in nonlinear control."

The recent success of this research effort in the solution of the nonlinear regulator problem relies heavily on our earlier AFOSR sponsored research on the problem of nonlinear feedback stabilization. In fact, in 1987, the Co-PI, Alberto Isidori, the principal investigator was honored as a Fellow of the IEEE for his contributions to the foundations of nonlinear control, an honor limited to a small fraction of the IEEE membership. In 1989, Professor Byrnes was also honored as a Fellow of the IEEE for his contribution to feedback stabilization and the control of linear and nonlinear systems. The research on the nonlinear regulator problem, and an application to hover control for a simplified model of a vertical take-off and landing aircraft, were highlighted in the 1991 publication of the AFOSR Research Accomplishments reviews. Since that time, the foundational paper on output regulation, which was published in the IEEE Transactions on Automatic Control was nominated for the George Axelby Award for the best paper published in the Transactions in the years 1989-1990, an award the paper subsequently won in December 1991 at the IEEE Conference on Decision and Control in Brighton, England. More recently, C.I. Byrnes and A. Isidori won the IFAC Automatica Best Paper Award for best paper published in the period 1990-1992. The winning paper was *"On the Attitude Stabilization of Rigid Spacecraft,"* Automatica, *27 (1991) 87-95.* and the award was presented at the opening ceremony of the IFAC World Congress in Sydney, Australia, July 1993.